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Classical aspects of quantum turbulence

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Abstract. Turbulence in helium II manifests itself as a disordered tangle of quantized vortex lines. The study of this form of disorder near absolute zero began with Vinen's heat-transfer experiments in the mid-1950s and is still a fertile ground of investigation. This article reviews the recent developments, which show surprising similarities between helium II turbulence and classical turbulence, and points to new directions of research.

1. Introduction and background

The study of quantized vortex lines in the turbulent flow in helium II was pioneered by Joe Vinen with a remarkable series of experiments in the mid-1950s (Vinen 1957a, b, c, d). More than forty years later this problem is still a fruitful area of investigation. New issues are being addressed as regards the relation between traditional, classical turbulence and quantum turbulence in helium II, and the nature of the disorder and dissipation at temperatures near absolute zero. The aim of this paper is to review the current work, point to new directions of investigations and stress the surprising similarities which are emerging between quantum turbulence and classical turbulence.

On the experimental side, the current activity includes the work done by Donnelly's group at the University of Oregon (Smith *et al* 1993), by McClintock's group at the University of Lancaster (Hendry *et al* 1994) and by Tabeling's group at the ENS Paris (Belin *et al* 1996, Maurer and Tabeling 1998). On the theoretical side, there are attempts to tackle the problem using either the vortex dynamics simulation approach (Barenghi *et al* 1997), or tools such as the condensate model (Nore *et al* 1997) which go beyond the traditional context of Landau's two-fluid hydrodynamics. The renewed interest in the problem was highlighted by the recent workshop on *Superfluid Turbulence and Cryogenics Probes* held at the University of Oregon in June 1998.

Landau's two-fluid theory provides the basic picture for studying the hydrodynamics of helium II. Landau's theory models helium II as an intimate mixture of two fluid components, the normal fluid and the superfluid. The superfluid is related to the quantum ground state, has density ρ_s and velocity $\boldsymbol{v}_s$, and flows without any friction. The normal fluid is related to the thermal excitations (phonons and rotons), has density ρ_n and velocity $\boldsymbol{v}_n$, and carries the entropy S and viscosity η of the entire liquid. The total density of helium II is $\rho = \rho_s + \rho_n$. The relative proportion of normal fluid and superfluid depends on the absolute temperature T. At $T = 0$, helium II is entirely superfluid ($\rho_s/\rho = 1$). If we increase T, the relative proportion of superfluid decreases, until, at the temperature of the lambda transition ($T = T_\lambda = 2.17$ K at saturated vapour pressure), helium II becomes entirely 'normal' ($\rho_n/\rho = 1$) and we are left with helium I, which is a classical, ordinary fluid.

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Although it is the ability to flow without any friction which gives the superfluid its name, it is the appearance of dissipation in the form of superfluid vortex lines which makes the flow of helium II so interesting (Donnelly 1991a). The two most important physical features of vortex lines—the quantization of the circulation and the mutual friction—are both associated with the work of Vinen (1957a, b, c, d, 1961). The quantization of the circulation is expressed by the condition

$$\int_{\mathcal{C}} \boldsymbol{v}_s \cdot \mathrm{d}\boldsymbol{l} = \Gamma \tag{1}$$

where $\mathcal{C}$ is a path around the vortex core, $\Gamma = h/m = 9.97 \times 10^{-4}\ \mathrm{cm}^2\ \mathrm{s}^{-1}$ is the quantum of circulation, h is Planck's constant and m is the mass of the helium atom. The mutual friction is a force which couples the normal-fluid and the superfluid components, and arises because the vortex lines scatter the phonons and rotons which constitute the normal fluid (Samuels and Donnelly 1990).

There are many ways to produce vortex lines. Vortex lines appear, for example, when a container filled with helium II is set into rotation. In this case the vortex configuration is ordered: the vortices align along the axis of rotation and the flow of helium II is laminar. There are also situations in which the vortices form a disordered, turbulent *tangle*: in this case we have *quantum turbulence*, also called *superfluid turbulence* in the literature.

Following Vinen's initial work, the early studies of the vortex tangle were carried out in a flow configuration related to heat transfer called *counterflow*, in which an opposite motion of superfluid and normal fluid is generated by an applied heat flux. Counterflow has no direct analogy in classical fluid mechanics, but is important in engineering applications when helium II is used as a coolant. The most recent work on the vortex tangle is more concerned with turbulent flows studied in classical fluid dynamics: in these flows turbulence is created by moving grids, propellers or blades. To stress the difference from counterflow, these more classical flows are referred to as *coflows*.

2. Counterflow turbulence

Counterflow is important because it is the prototype of heat transfer and is associated with engineering applications. Consider a channel which is closed at one end and is open to the helium bath at the other end. At the closed end a resistor dissipates a known heat flux W. In an ordinary fluid, provided that one is careful to prevent convective motion, heat is transferred from one end to the other by thermal conduction; the heat flux W is proportional to the temperature gradient ∇T and there is a well defined thermal conductivity k at small W, $W = k\,\nabla T$. The situation is very different in helium II: here the heat is carried by the normal fluid away from the heater, $W = \rho S T v_n$, where S is the entropy per unit mass. Because of the closed end, the mass flux $\boldsymbol{j} = \rho_s \boldsymbol{v}_s + \rho_n \boldsymbol{v}_n$ is zero and some superfluid must flow towards the heater to conserve mass, $\boldsymbol{v}_s = -(\rho_n/\rho_s)\boldsymbol{v}_n$. In this way a relative *counterflow* velocity $V = |\boldsymbol{v}_n - \boldsymbol{v}_s|$ between the normal fluid and the superfluid is set up which is proportional to the applied heat flux W, $V = W/(\rho_s S T)$.

Vinen discovered that if the heat flux W is increased, the relative motion of normal fluid and superfluid becomes stronger until a critical velocity V_{c1} is reached (Vinen 1957a, b, c, d). At this point the perfect ability of helium II to transfer heat breaks down: a vortex tangle appears and introduces dissipation. The tangle, which is usually observed by measuring the attenuation of second sound, can be characterized by its line density L_0, which is the length of vortex line per unit volume. Vinen's measurements showed that for $V > V_{c1}$ the relation between L_0 and V^2 is linear:

$$L_0 \approx \gamma V^2 \tag{2}$$

where γ is a temperature-dependent parameter. Geometrically, $L_0^{-1/2}$ represents the average spacing between the vortices in the tangle. Vinen also derived a simple phenomenological equation for L_0 which yields (2) as solution, although the numerical value of the parameter γ was left undetermined.

On the theoretical side the numerical simulations of Schwarz (1982, 1985, 1988) shed much light onto the problem. After assuming for simplicity that $V = |\boldsymbol{v}_n - \boldsymbol{v}_s|$ is a constant, Schwarz showed that, provided that $V > V_{c1}$, a self-sustaining vortex tangle driven by V follows from the simple rules of vortex dynamics. The calculation also yielded the quantity γ without any adjustable parameters.

Following Vinen's work, a great number of experiments were performed by other investigators, above all by Brewer and Edwards (1962), van Beelen and co-workers (e.g. Slegtenhorst *et al* 1981), Donnelly and co-workers (e.g. Barenghi *et al* 1982, Swanson *et al* 1983, Donnelly 1993) and Tough and co-workers (e.g. Ladner *et al* 1976). A major difficulty soon arose because γ seemed to vary greatly from experiment to experiment. A very detailed analysis of all of the data available was carried out by Tough (1987), who discovered the existence of separate turbulent states characterized by different values of γ. Tough found that in channels of circular or almost square cross section at increasing values of $V > V_{c1}$, there is first a regime of moderate vortex line density, which he called the *T-1 turbulent state*. If $V = V_{c2}$, where V_{c2} is a second critical velocity, the vortex line density L_0 becomes suddenly larger (Martin and Tough 1983); the new regime for $V > V_{c2}$ was called the *T-2 turbulent state* by Tough. The lower-density state, T-1, is absent in high-aspect-ratio (rectangular) channels like Vinen's, in which the line density has the same value as in the T-2 state. The value of γ in the T-2 state corresponds to what was found numerically by Schwarz.

For many years the nature of the two turbulent states and the physical meaning of the transition at $V = V_{c2}$ in circular channels have been a mystery. The puzzle was solved recently by Melotte and Barenghi (1998), who addressed for the first time the issue of the velocity profile of the normal fluid and its stability. They started with the observation that the normal fluid must obey the same no-slip boundary conditions as a classical fluid. Hence, for $V < V_{c1}$ in the laminar counterflow regime (before the tangle appears), the profile of the normal fluid is $\boldsymbol{v}_n = V_0(1 - r^2)\hat{\boldsymbol{z}}$ like classical Poiseuille pipe flow, where r is the radial variable, V_0 is some constant proportional to the applied heat flux W and $\hat{\boldsymbol{z}}$ is the unit vector in the axial direction along the channel. Melotte and Barenghi then argued that for $V_{c1} < V < V_{c2}$, in the weak T-1 turbulent regime, the normal fluid is still laminar, the parabolic profile being only mildly flattened near the centre of the channel by the mutual friction between the normal fluid and the vortex tangle of density L_0. Finally they computed the stability of $\boldsymbol{v}_n$ with respect to infinitesimal perturbations of the form $\mathrm{e}^{\mathrm{i}kz+\mathrm{i}m\phi}$ where ϕ is the azimuthal angle, and k and m are respectively the axial and azimuthal wavenumber. The calculation showed that if L_0 is large enough, then $\boldsymbol{v}_n$ becomes unstable. The computed critical velocities agree very well with the observations of the T-1–T-2 transition in the ranges of temperature and channel size observed. Because of the mathematical similarity between their calculation and the classical problem of Osborne Reynolds of the stability of pipe flow, Melotte and Barenghi argued that the T-1–T-2 transition corresponds to the transition to turbulence of the normal fluid. This conjecture is reinforced by the observation that in their calculation a very wide range of wavenumbers become excited just above the computed instability: this suggests that the flow which emerges from the instability has a great spatial complexity, and hence is likely to be turbulent.

The picture which appears from this work is thus the following: in the T-1 state ($V < V_{c1}$)

the superfluid is turbulent (there is a vortex tangle) but the normal fluid is still laminar; in the T-2 state the normal fluid becomes turbulent too—hence the increased dissipation and higher vortex line density. This scenario is consistent with the agreement between Schwarz's calculation and the measurements for the T-2 state: once v_n is turbulent, it can be approximated fairly well by a time-averaged, uniform profile, which is the assumption originally made for simplicity by Schwarz.

3. Coflow turbulence

Unlike counterflow turbulence, coflow turbulence in helium II has a direct analogy with classical turbulence. In classical fluid mechanics the turbulence intensity is measured by the *Reynolds number*, $Re = UL/\nu$, where U and L are length scales and $\nu = \eta/\rho$ is the kinematic viscosity. Recent experiments show that at high Reynolds numbers the turbulent coflow motion of helium II is very similar to classical turbulence. The evidence is robust. The experiments are the following.

- Mass flow rates and pressure drops along pipes at $Re \approx 10^6$ can be well described by using the classical relations for high-Reynolds-number classical flows (Walstrom *et al* 1988).
- Experiments on large-scale turbulent vortex rings at $Re \approx 4 \times 10^4$ detected normal-fluid vorticity and superfluid vorticity moving together as a single structure (Borner *et al* 1983, Borner and Schmidt 1985).
- Experiments on turbulent Taylor–Couette flow between concentric rotating cylinders at $Re \approx 4 \times 10^3$ showed the typical structures of classical turbulent Taylor–Couette flow (Bielert and Stamm 1993).
- Experiments on the decay of superfluid vorticity created by towing a grid in a sample initially at rest showed that the vorticity decays in time according to the same laws as of the decay of classical turbulence (Smith *et al* 1993). More surprising, the decay appears independently of temperature, from T_λ down to the lowest temperature measured, 1.4 K (Stalp 1998a, b).
- Experiments on turbulence created by rotating blades by Maurer and Tabeling (1998) showed the same Kolmogorov $k^{-5/3}$-energy spectrum of classical turbulence for both helium I and helium II, at temperature as low as $T = 1.4$ K.

The lack of temperature dependence of the last two experiments is remarkable: the normal fluid alone cannot be held responsible for helium's classical behaviour at $T = 1.4$ K: at this temperature ρ_n/ρ is only 7% and one expects the normal fluid to be dynamically unimportant.

The classical behaviour of turbulent helium II is at first surprising, because we are historically accustomed to the idea that the flow of helium II is very different from the flow of a classical fluid: traditional examples are second sound, thermal counterflow, superleaks, etc. Even the more complex vortex flows which have been studied recently, such as Couette flow, confirm the non-classical motion of helium II (Barenghi 1992, Henderson *et al* 1995, Barenghi 1997, Henderson and Barenghi 1998). But all of these flows (with and without vortices) refer to laminar flows at rather low Reynolds numbers: the key distinction which must be made is thus between *laminar* and *turbulent* flows of helium II.

To explain the observations it has been suggested (Donnelly 1991b) that in the turbulent regime, when there is a large density of vortex lines, the superfluid and the normal fluid *lock together* and helium II behaves classically like a single fluid of density ρ, a concept referred to as *vortex-coupled superfluidity*.

An important observation as regards understanding vortex-coupled superfluidity was first made by Samuels (1993), namely that vorticity in the normal fluid has a significant effect on the quantized vortex lines. To pursue this idea, one must follow the vortex dynamics method pioneered by Schwarz (1982, 1985, 1988) to study the vortex tangle of counterflow turbulence. The method is the following. An arbitrary configuration of vortex lines is discretized into a number N of points, and the position of each point is integrated over time using the equation

$$\frac{\mathrm{d}\boldsymbol{s}}{\mathrm{d}t} = \boldsymbol{v}_i + \alpha \boldsymbol{s}' \times (\boldsymbol{v}_n - \boldsymbol{v}_i). \tag{3}$$

Here $\boldsymbol{s} = \boldsymbol{s}(t, \xi)$ is the position of a vortex point, ξ is the arc length, t is the time, a prime denotes differentiation with respect to ξ, $\alpha = \rho_n B/(2\rho)$, B is the known temperature-dependent mutual friction coefficient (Barenghi *et al* 1983) and the self-induced velocity $\boldsymbol{v}_i$ is determined by the Biot–Savart integral

$$\boldsymbol{v}_i(\boldsymbol{s}) = \frac{\Gamma}{4\pi} \int_{\mathcal{V}} \frac{(\boldsymbol{z} - \boldsymbol{s}) \times \mathrm{d}\boldsymbol{z}}{|\boldsymbol{z} - \boldsymbol{s}|^3} \tag{4}$$

which must extend over the entire vortex configuration in the volume $\mathcal{V}$. Note that in writing (3) the small transverse part of the mutual friction has been neglected. The computer code which performs this calculation must allow for a variable number N of points (more points are required if a kink develops along a vortex line) and for vortex reconnections, whose existence was demonstrated by Koplik and Levine (1993) using the condensate model (see section 3).

To investigate the possibility that the two fluids lock together, Barenghi *et al* (1997) performed the following numerical simulation, which generalizes the early work of Samuels (1993). They started with the observation that in the experiments under consideration the Reynolds number is so high that the normal fluid must be turbulent. It is known from the numerical simulations of classical turbulence (She *et al* 1990, Vincent and Meneguzzi 1994, Siggia 1981, Kerr 1985) and from experiments (Douady *et al* 1991) that turbulence is not a uniform randomness: regions of intense, concentrated vorticity called *vortex tubes* appear spontaneously in the flow, move about and disappear after a certain lifetime. Therefore vortex tubes must be present in the turbulent normal fluid. To model these structures, Barenghi *et al* chose an Arnold–Beltrami–Childress (ABC) flow $\boldsymbol{v}_n = (u_n, v_n, w_n)$, whose components in Cartesian coordinates are given by

$$u_n = A \sin(2\pi z/\lambda) + C \cos(2\pi y/\lambda) \tag{5}$$

$$v_n = B \sin(2\pi x/\lambda) + A \cos(2\pi z/\lambda) \tag{6}$$

$$w_n = C \sin(2\pi y/\lambda) + B \cos(2\pi x/\lambda). \tag{7}$$

Here λ is a length scale and A, B and C are parameters. ABC flows (Dombre *et al* 1986) are a convenient way to model regions of concentrated vorticity and provide a one-scale model of normal-fluid turbulence. They are solutions of the steady Euler equation and of the time-dependent, forced Navier–Stokes equation. Despite the apparent simplicity, their streamlines have a complex Lagrangian pattern which includes chaotic particle paths at certain values of the parameters. ABC flows have also been used to study turbulent processes of dynamo action in magneto-hydrodynamics (Gilbert and Childress 1995, Galloway and Proctor 1992). Finally, ABC flows have non-zero helicity (Moffatt 1969), a property which has been associated with turbulence structures both in experiments and numerical simulations (Moffatt and Tsinober 1992, Kit *et al* 1987).

The numerical simulation of Barenghi *et al* calculated the time evolution of an arbitrary initial configuration of superfluid vortex lines in the presence of a driving ABC flow. The calculation was performed inside a three-dimensional periodic box of size λ. Typically the calculation started with an initial vortex ring. Under the influence of the normal flow, the ring became unstable and distorted, the total length of vortex line increased and a vortex tangle developed. But the vortex tangle was very different from the almost isotropic and homogeneous tangle obtained by Schwarz (1982, 1985, 1988) in counterflow turbulence. Barenghi *et al* found that bundles of superfluid vortex lines were created and concentrated in the regions where the vorticity of the normal fluid is high. The physical mechanism behind the creation of these bundles is the instability of a superfluid helical vortex wave in the presence of normal flow parallel to the line (Ostermeier and Glaberson 1975). As vortex waves become unstable and grow, more line length is created, hence more vortex loops exhibit the same instability, and so on, until non-linear effects (the Biot–Savart law and reconnections) saturate the growth. Although the *microscopic* superfluid velocity pattern in the bundles is very complicated, its *macroscopic* average over a region larger than the intervortex separation is found to be similar to the vorticity field of the normal fluid. This *vorticity matching* is consistent with the observations. Numerical investigation of the growth timescale for the vortex bundles showed that it is of the same order as the ABC flow timescale; since the lifetime of the vortex tubes observed in turbulence is of the order of few turnover times, one concludes that there is enough time for the vorticity-matching process to take place.

The ABC model is clearly too simple for use in making a direct quantitative comparison with the experiments, but it confirms the locking mechanism which has been postulated to explain the observations and provides a physical explanation for this mechanism.

4. Turbulence at absolute zero

The experiments described above motivated Nore *et al* (1997) to perform a calculation based on the Gross–Pitaevskii equation for a Bose–Einstein condensate. This is a simplified model of superfluidity at $T = 0$ which naturally yields vortex lines and vortex reconnections (Koplik and Levine 1993). The governing equation is the non-linear Schrödinger equation

$$\mathrm{i}\hbar\frac{\partial\psi}{\partial t} = -\frac{\hbar^2}{2m}\nabla^2\psi - \mu\psi + U\psi|\psi|^2 \tag{8}$$

where ψ is the condensate's wave function, μ is the chemical potential and U is the strength of the delta-function repulsion interaction between the bosons. Nore *et al* took as an initial condition a convenient Taylor–Green vortex flow and computed the time evolution of the vortex tangle. Expressed as a velocity $\boldsymbol{v}_s = (u_s, v_s, w_s)$ via a Madelung transformation, this initial condition is

$$u_s = \sin(x)\cos(y)\cos(z) \tag{9}$$

$$v_s = -\cos(x)\sin(y)\cos(z) \tag{10}$$

$$w_s = 0. \tag{11}$$

Physically, the Taylor–Green vortex is related to the flow between two counter-rotating disks. Nore *et al* found that the energy spectrum of the kinetic energy of the tangle is consistent with the classical Kolmogorov $k^{-5/3}$-law, in agreement with the experimental finding of Tabeling. The appearance of Kolmogorov's law shows that there is great similarity in the way the energy is transferred from large scales (small wavenumbers k) to small scales (large wavenumbers k) in the two problems of classical turbulence and turbulence of a Bose–Einstein condensate, which are at first sight unrelated. Nore *et al* also found that the incompressible kinetic energy of the decaying vortex tangle is converted into sound waves. This observation is consistent with an early calculation of Jones and Roberts (1982) concerning the fate of a shrinking vortex ring. In the case of zero-temperature turbulence, the generation of sound is important because it is the only mechanism to dissipate kinetic energy.

It must be noticed that one does not need to reach $T = 0$ for the Gross–Pitaevskii equation to be a valid model of superfluidity: because of the strong temperature dependence of ρ_n and ρ_s, at about $T = 0.6$ K there is already virtually no normal fluid left, and helium II is almost entirely a pure superfluid. Moreover, at this temperature the contribution of the rotons to the energy spectrum becomes less important than the phonons': this is important for the validity of the condensate model, as the spectrum of the Gross–Pitaevskii equation does not have a roton minimum. From this point of view, therefore, the experiment just started by McClintock's group at the University of Lancaster is an interesting development: the aim of the experiment is to generate a turbulent vortex tangle by oscillating a grid at temperatures of about 70 mK. Although there are no results yet, this experiment is in the low-temperature limit for which the model of Nore *et al* (1997) applies, and adds to the number of experiments currently in progress to study the turbulence of helium II.

A difficulty with low-temperature turbulence experiments is that the usual second-sound technique used to detect vortex lines does not work. Traditionally, the only technique still available consists in detecting ions trapped by the vortex lines. Recently Samuels and Barenghi (1998) have pointed out that at these low temperatures one could use thermometers to detect the vortices, as their kinetic energy is transformed into sound, and hence into phonons and heat.

5. Discussion and forward look

It is apparent from this selected review of the study of helium II turbulence, as it has developed from the pioneering work of Vinen until now, that this topic, traditionally studied by the condensed matter physicists, now has many important points in common with the classical turbulence studied by fluid dynamicists and applied mathematicians.

Because of the quantization of the circulation and the smallness of the vortex core ($\approx 10^{-8}$ cm), quantized vortex lines in helium II are Nature's best realizations of the vortex filaments of an Euler fluid, which have been studied by fluid dynamicists and applied mathematicians since the times of Lord Kelvin. From this point of view, a turbulent tangle of vortex lines in helium II is an idealized but mathematically well defined toy model of turbulence. In classical turbulence the eddies can have a huge range of size and strength, while the superfluid eddies (the vortex lines) have the same strength, due to the quantization of the circulation. For example, vortex tangles are an ideal ground on which to test ideas about the *topological complexity of turbulent flows*. The reason for this is that vortex lines in helium II are always geometrically well defined, unlike what happens in the numerical calculations of classical turbulence, in which the numerical noise of regions of weak vorticity limits the definition of vortex lines. Topological fluid dynamics (Moffatt 1969, Ricca and Berger 1996) is a growing research area—see the recent work of Ricca *et al* (1998) on vortex knots. This new interest has been highlighted by the decision of the Isaac Newton Institute for Mathematical Sciences in Cambridge to devote an entire semester to this topic in the year 2000.

The more outstanding link between quantum and classical turbulence is the recent discovery that, unlike what happens at small Reynolds numbers, the high-Reynolds-number turbulent motion of helium II is similar to classical turbulence. The experimental evidence is robust, and appears to be independent of temperature within the range explored. Clearly much work is needed, both theoretically and experimentally, to find the limits of validity of this observation in terms of temperatures, vorticity intensity, timescales and length scales. If the limits are large, then the fascinating possibility is raised of probing turbulent vorticity using second sound when studying issues of classical turbulence. In classical fluid dynamics, in fact, one can measure the velocity well (for example using the Doppler technique) but it is difficult to reconstruct the vorticity, which is the quantity more of interest when studying turbulence; for helium II, on the contrary, second sound detects the vorticity directly.

From a theoretical point of view, it is clear that much more attention must be paid to the normal fluid. The calculations of Samuels (1993) and of Barenghi *et al* (1997) give strong support to the idea of vorticity matching and provide a physical mechanism, but they suffer from an important limitation: there is no back-reaction of the vortex tangle onto the normal fluid—that is to say, $\boldsymbol{v}_n$ is fixed *a priori*. The same limitation appears in all other calculations which made use of the vortex dynamics approach: they all determined the vortex tangle as a function of an assigned $\boldsymbol{v}_n$, either constant (Schwarz 1982, 1985, 1988), or Poiseuille (Samuels 1992, Aarts and deWaele 1994) or a single vortex (Samuels 1993) or ABC flow (Barenghi *et al* 1997). Essentially, all of these calculations were only *kinematic*. What is necessary is a new *dynamically self-consistent* calculation, in which the normal fluid, forced by the mutual friction, evolves self-consistently alongside the vortex tangle. Only a self-consistent approach will clarify issues such as the validity and the range of the phenomenon of vorticity matching, in which the two fluids are so strongly coupled by mutual friction that helium II behaves like a single fluid of density ρ. This is a non-trivial computational task, however, because it combines the difficulty of the numerical calculations of classical turbulence with the vortex tangle calculation, both in three dimensions.

Finally, the (experimental) observation of the classical Kolmogorov spectrum for helium II at the temperatures observed, from T_λ down to 1.4 K, and the (theoretical) observation of the same spectrum in turbulence at $T = 0$ clearly raises issues about the meaning and generality of the $k^{-5/3}$-law. Why is quantum turbulence so classical? Attempts to answer this question will surely shed more light on the nature of classical turbulence too.

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References

Aarts R G K and deWaele A T A M 1994 *Phys. Rev.* B **50** 10 069
Barenghi C F 1992 *Phys. Rev.* B **45** 2290
Barenghi C F 1997 *Phil. Trans. R. Soc.* **355** 2025
Barenghi C F, Donnelly R J and Vinen W F 1983 *J. Low Temp. Phys.* **52** 189
Barenghi C F, Samuels D C, Bauer G H and Donnelly R J 1997 *Phys. Fluids* **9** 2631
Barenghi C F, Swanson C E and Donnelly R J 1982 *Phys. Rev. Lett.* **48** 1187
Belin F, Maurer J, Tabeling P and Willaime H 1996 *J. Physique* II **6** 573
Bielert F and Stamm G 1993 *Cryogenics* **33** 938
Borner H, Schmeling T and Schmidt D W 1983 *Phys. Fluids* **26** 1410
Borner H and Schmidt D W 1985 *Flow of Real Fluids (Springer Lecture Notes in Physics 235)* (Berlin: Springer) p 135
Brewer D F and Edwards D O 1962 *Phil. Mag.* **7** 721
Dombre T, Frisch U, Greene J M, Henon M, Mehr A and Soward A M 1986 *J. Fluid Mech.* **167** 353
Donnelly R J 1991a *Quantized Vortices in Helium II* (Cambridge: Cambridge University Press)
Donnelly R J 1991b *High Reynolds Number Flows Using Liquid and Gaseous Helium* (Berlin: Springer)
Donnelly R J 1993 *Annu. Rev. Fluid Mech.* **25** 325
Douady S, Couder Y and Brachet M E 1991 *Phys. Rev. Lett.* **67** 983
Galloway D J and Proctor M R E 1992 *Nature* **356** 691
Gilbert A D and Childress S 1995 *Stretch, Twist, Fold, the Fast Dynamo* (Berlin: Springer)
Henderson K L and Barenghi C F 1998 submitted

Henderson K L, Barenghi C F and Jones C A 1995 *J. Fluid Mech.* **283** 329
Hendry P C, Lawson N S, Lee R A M and McClintock P V E 1994 *Nature* **368** 315
Jones C A and Roberts P H 1982 *J. Phys. A: Math. Gen.* **15** 2599
Kerr R M 1985 *J. Fluid Mech.* **153** 31
Kit E, Tsinober A and Balint J L 1987 *Phys. Fluids* **30** 3328
Koplik J and Levine H 1993 *Phys. Rev. Lett.* **71** 1375
Ladner D R, Childers R K and Tough J T 1976 *Phys. Rev.* B **13** 2918
Martin K P and Tough J T 1983 *Phys. Rev.* B **27** 2788
Maurer J and Tabeling P 1998 *Europhys. Lett.* **43** 29
Melotte D J and Barenghi C F 1998 *Phys. Rev. Lett.* **80** 4181
Moffatt H K 1969 *J. Fluid Mech.* **36** 117
Moffatt H K and Tsinober A 1992 *Annu. Rev. Fluid Mech.* **24** 281
Nore C, Abid M and Brachet M E 1997 *Phys. Rev. Lett.* **78** 3896
Ostermeier R M and Glaberson W I 1975 *J. Low Temp. Phys.* **21** 191
Ricca R L and Berger M A 1996 *Phys. Today* **49** 24
Ricca R L, Samuels D C and Barenghi C F 1998 *Advances in Turbulence* ed U Frisch (Dordrecht: Kluwer)
Samuels D C 1992 *Phys. Rev.* B **96** 11 714
Samuels D C 1993 *Phys. Rev.* B **47** 1107
Samuels D C and Barenghi C F 1998 *Phys. Rev. Lett.* at press
Samuels D C and Donnelly R J 1990 *Phys. Rev. Lett.* **65** 187
Schwarz K W 1982 *Phys. Rev. Lett.* **49** 283
Schwarz K W 1985 *Phys. Rev.* B **31** 5782
Schwarz K W 1988 *Phys. Rev.* **38** 2398
She Z S, Jackson E and Orszag S A 1990 *Nature* **344** 226
Siggia E D 1981 *J. Fluid Mech.* **107** 375
Slegtenhorst R P, Marees G and van Beelen H 1981 *Physica* B **113** 367
Smith M R, Donnelly R J, Goldenfeld N and Vinen W F 1993 *Phys. Rev. Lett.* **71** 2583
Stalp S 1998a *PhD Thesis* University of Oregon
Stalp S 1998b private communication
Swanson C E, Barenghi C F and Donnelly R J 1983 *Phys. Rev. Lett.* **50** 190
Tough J T 1987 *Progress in Low Temperature Physics* vol VIII, ed D F Brewer (Amsterdam: North-Holland) p 133
Vincent A and Meneguzzi M 1994 *J. Fluid Mech.* **258** 245
Vinen W F 1957a *Proc. R. Soc.* A **240** 114
Vinen W F 1957b *Proc. R. Soc.* A **240** 128
Vinen W F 1957c *Proc. R. Soc.* A **242** 493
Vinen W F 1957d *Proc. R. Soc.* A **243** 400
Vinen W F 1961 *Proc. R. Soc.* A **260** 218
Walstrom P L, Weisend J G, Maddocks J R and VanSciver S W 1988 *Cryogenics* **28** 101